

Chapter \Rightarrow 1

By \Rightarrow

Number System

Exercise \Rightarrow 1.1

1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where q and p are integers and $q \neq 0$.

Solution \Rightarrow Yes, zero is a rational number.
Because it is a form of $\frac{p}{q} = \frac{0}{1}$ Ans.

2. Find six rational numbers between 3 and 4.

Solution \Rightarrow $N = 6$ $(N+1) = 6+1 = 7$
 $\Rightarrow \frac{3 \times 7}{1 \times 7} = \frac{21}{7}$, $\frac{4 \times 7}{1 \times 7} = \frac{28}{7}$

Rational numbers between $\frac{21}{7}$ and $\frac{28}{7}$.

$\Rightarrow \frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$ Ans.

3. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Solution \Rightarrow $N = 5$ $(N+1) = 5+1 = 6$
 $\Rightarrow \frac{3 \times 6}{5 \times 6} = \frac{18}{30}$, $\frac{4 \times 6}{5 \times 6} = \frac{24}{30}$

Five rational numbers between $\frac{18}{30}$ and $\frac{24}{30}$.

$\Rightarrow \frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$ Ans

Q4. State whether the following statements are true or False. Give reason for your answer:

(i) Every Natural No is a whole No \Rightarrow True

Reason \Rightarrow Natural no started by $1, 2, \dots, \infty$ and whole no started with $0, 1, 2, \dots, \infty$

Hence, the natural no come in whole number.

ii) Every integer is a whole number \Rightarrow False

Reason \Rightarrow Integers $[-3, -2, -1, 0, 1, 2, 3, 4, \text{etc.}]$

Whole no $[0, 1, 2, 3, 4, 5, 6, 7, \text{etc.}]$

$-2 =$ Integer

by but $-2 \neq$ whole no

iii) Every rational numbers is a whole Number \Rightarrow False.

Reason \Rightarrow Rational Number $\frac{1}{2}$

\Rightarrow Whole no = $0, 1, 2, 3, 4, \text{etc}$

$\Rightarrow \frac{1}{2}$ rational No or by $\frac{1}{2} \neq$ whole No.

Exercise 1.2

Irrational Number

1. State whether the following statements are true or false.

or false. Justify your answer.

i) Every irrational Number is a real no.

Ans \Rightarrow It is true because the real no. made of two type of no. that are rational and irrational no. Hence every irrational no. is a real no.

ii) Every points on the number line is of the form \sqrt{m} , where m is a natural no.

Ans \Rightarrow False, no negative no. can be the square root of any natural no.

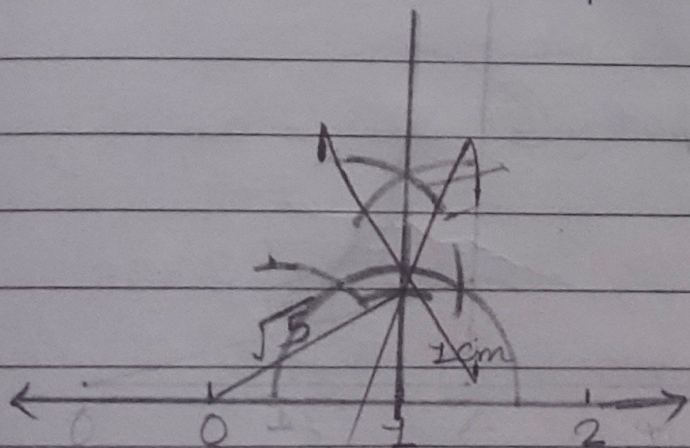
iii) Every real Number is an irrational No.

Ans \Rightarrow It is False. Because 2 is a real no. but not Irrational no.

2. Are the square roots of all positive integers irrational? If not, give an example of all degree square root of no. that is a rational?

Ans \Rightarrow No $\sqrt{4} = 2$ is a rational no.

Show how $\sqrt{5}$ can be represented on number line.



$$\begin{aligned} &\sqrt{5} \\ &\Rightarrow \sqrt{4+1} \\ &\Rightarrow \sqrt{(2)^2+(1)^2} \end{aligned}$$

$$H^2 = l^2 + b^2 \Rightarrow (OA)^2 = (AB)^2 + (BO)^2$$

$$\Rightarrow (OA)^2 = 1^2 + 2^2 \Rightarrow (OA)^2 = 1 + 4 \Rightarrow (OA)^2 = 5$$

$$(OA)^2 = 5$$

$$OA = \sqrt{5} \text{ Ans}$$

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② You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansion of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are without actually doing the long division? If so, how?

Solution: ① $\frac{1}{7} = 0.\overline{142857}$ Ans

② $\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857}$ Ans

③ $\frac{3}{7} = \frac{3}{7} \times \frac{1}{7} = 3 \times 0.\overline{142857}$ Ans

④ $\frac{4}{7} = \frac{4}{7} \times \frac{1}{7} = 4 \times 0.\overline{142857}$ Ans

⑤ $\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times 0.\overline{142857}$ Ans

⑥ $\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times 0.\overline{142857} = 0.\overline{857142}$ Ans

5 Express the following in the form $\frac{p}{q}$ where p and q are integers $q \neq 0$.

i) $0.\overline{6} \Rightarrow$ Solution: Let $x = 0.\overline{6666}$ — ①
by equation ① $\times 10$

$10x = 6.\overline{6666}$

Subtract equation ② from ①

$9x = 6.\overline{6666}$ — ②

$x = 0.\overline{6666}$

$9x = 6.0000$

$x = \frac{6}{9}$

$x = \frac{2}{3}$ Ans

ii) $0.\overline{47}$

Solution: Let $x = 0.\overline{4777}$

eg. $10x = 4.\overline{7777}$ ① \Rightarrow by equation (i) $\times 10$

$$\Rightarrow 100x = 47.7 \quad \text{--- (i)}$$

Subtract equation (ii) from (i)

$$1000x = 47.7777$$

$$10x = 4.7777$$

$$90x = 43.0000$$

$$x = \frac{43}{90} \quad \underline{\text{Ans}}$$

(iii) $0.\overline{001}$

Solution \Rightarrow Let $x = 0.001001001 \dots$ --- (i)

by equation (i) $\times 1000$

$$1000x = 1.001001 \dots \dots \dots \quad \text{--- (2)}$$

by equation $= \text{(ii)} - \text{(i)}$

$$1000x = 1.001001 \quad \text{--- (2)}$$

$$- x = 0.001001 \quad \text{--- (1)}$$

$$999x = 1.000000$$

$$x = \frac{1}{99} \quad \underline{\text{Ans}}$$

Q5. What can the maximum number of digits be in the repeating block of digits in decimal expansion of $\frac{1}{17}$. of a perform the division to check answer.

Solve \rightarrow

$$17 \overline{) 100} \quad 0.0588235294117647$$

85	
150	
136	
140	
136	
240	
34	
260	
51	
90	
85	
50	
34	
160	
153	
70	
68	

\rightarrow

$\times 20$	17
$\times 30$	17
	130
	119
$\times 110$	102
$\times 80$	69
$\times 120$	119

24. Express $0.9999\dots$ in the form $\frac{p}{q}$. Are you helped by your answer? Discuss with your teachers and classmate's discuss why the answer makes sense.

Solution: 0.9

Let $x = 0.9999\dots$ — (i)

by equation (i) $\times 10$

$10x = 9.9999\dots$ — (ii)

by equations (ii) - (i)

$10x = 9.99999$

0.99999

$9x = 9.000000$

$x = \frac{9}{9}$ $x = 1$ Ans

6. Look at several examples of rational numbers in the form $\frac{p}{q}$ ($q \neq 0$) where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansion). Can you guess what property q must satisfy?

Solution: $\frac{1}{2} = 0.5$
 $\frac{1}{4} = 0.25$

$\frac{1}{5} = 0.2$
 $\frac{1}{25} = 0.04$

7. Write three no. whose decimal expansion are non-terminating non-recurring.

Question: Irrational No \rightarrow $\sqrt{\text{Imperfect Square}}$
 $\Rightarrow \sqrt{2} \Rightarrow 1.4142135\dots$
 $\Rightarrow \sqrt{3} \Rightarrow 1.7320508075\dots$
 $\Rightarrow \sqrt{5} \Rightarrow 2.2360679\dots$

Q8. Find three different irrational no between the rational no

$\frac{5}{7}$ and $\frac{9}{11}$

Solution: $0.714285 \dots, \dots, \frac{9}{11} = 0.\overline{81}$

0.72387

0.733842

0.745976

Q9. Classify the following no as ^{or} rational or irrational.

(i) $\sqrt{23} \Rightarrow \sqrt{23}$ is irrational No Ans

(ii) $\sqrt{225} \Rightarrow \sqrt{15 \times 15} = \frac{15}{1} =$ rational no Ans

(iii) 0.3796

Solution $\Rightarrow 0.3796$ is terminating
Hence this no is a rational no.

(iv) 7.478478

Solution $\Rightarrow 7.478478$ non-terminating repeating. Hence these no are rational no.

(v) $1.101001000100001\dots$

Solution $\Rightarrow 1.101001000100001\dots$ is a non-terminating non-recurring. Hence these no are irrational no.

Exercise 1.5

Rules \Rightarrow (i) Negative of an irrational no is an irrational no.

(ii) The sum or difference of irrational numbers and rational number is an irrational number.

(iii) The product of a non-zero rational number and an irrational number is an irrational number.

(iv) The sum, difference, product and quotient two irrational numbers need not be an irrational number.

1. Classify the following numbers as rational or irrational.

a) $2 - \sqrt{5}$

\Rightarrow 2 is rational no and $\sqrt{5}$ is an irrational no and the difference of the rational no and an irrational no is Irrational. No.

b) $(3 + \sqrt{23}) - \sqrt{23}$

$\Rightarrow 3 + \sqrt{23} - \sqrt{23}$

\Rightarrow 3 is a rational number.

c) $\frac{2\sqrt{3}}{7\sqrt{7}}$

\Rightarrow Solution $\Rightarrow \frac{2}{7} \frac{\sqrt{7}}{\sqrt{7}}$ $\frac{2}{7}$ is a rational no.

d) $\frac{1}{\sqrt{2}} = 1 \div \sqrt{2}$

\Rightarrow 1 is a rational no and $\sqrt{2}$ is an irrational no. This is irrational No.

(C) 2π
 \Rightarrow 2 is a rational no and π is irrational. Hence the product of rational no and irrational no is irrational.

(F) $2\pi r$
Solution \Rightarrow Circumference of circle = $2\pi r$
 $\Rightarrow C = \pi d$
 $\Rightarrow r = \frac{C}{2\pi}$

\Rightarrow So, $\frac{C}{d}$ is a rational but if we calculate $\Rightarrow \frac{C}{d}$ it gives π

\Rightarrow π is a irrational no. Quotient of two irrational no is always irrational so $r \neq \frac{C}{d}$.

(4) Represent $\sqrt{9.3}$ on the number line.

Solution:- On the second page

(5) Rationalise the denominator of the following.

(1) $\frac{1}{\sqrt{7}}$ Solution: $\frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{(\sqrt{7})^2} = \frac{\sqrt{7}}{7}$ Ans

2. Simplify each of the following expression.

(3 + $\sqrt{3}$) (2 + $\sqrt{2}$)
 $\Rightarrow 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$ Ans

(3 + $\sqrt{3}$) (3 - $\sqrt{3}$)
 $a^2 - b^2 = (a+b) \cdot (a-b)$
 $= 3^2 - (\sqrt{3})^2$

⇒ 9-3

⇒ 6 Ans

ii) $(\sqrt{5} + \sqrt{2})^2$
 ⇒ $(\sqrt{5})^2 + (\sqrt{2})^2 + 2 \times \sqrt{5} \times \sqrt{2}$
 ⇒ $5 + 2 + 2 \times \sqrt{10}$
 ⇒ $7 + 2\sqrt{10}$ Ans

iii) $(\sqrt{5} - \sqrt{2}) \cdot (\sqrt{5} + \sqrt{2})$
 ⇒ $(a+b)(a-b) = a^2 - b^2$
 $(\sqrt{5})^2 - (\sqrt{2})^2$
 = $5 - 2$
 = 3 Ans

Recall π is defined as the ratio of the circumference (say c) of a circle diameter (say d) that is $\pi = \frac{c}{d}$ this seems to contradict the fact, the π is irrational. How will you resolve this contradiction.

i) $\frac{1}{\sqrt{7} - \sqrt{6}} \Rightarrow \frac{1}{\sqrt{7} - \sqrt{6}} \times \frac{\sqrt{7} + \sqrt{6}}{\sqrt{7} + \sqrt{6}} = \frac{\sqrt{7} + \sqrt{6}}{(\sqrt{7})^2 - (\sqrt{6})^2}$

$\sqrt{7} + \sqrt{6}$ Ans

$\frac{1}{\sqrt{5} + \sqrt{2}}$

⇒ $\frac{1}{\sqrt{5} + \sqrt{2}} \times \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} - \sqrt{2}} = \frac{\sqrt{5} - \sqrt{2}}{(\sqrt{5})^2 - (\sqrt{2})^2}$

⇒ $\frac{\sqrt{5} - \sqrt{2}}{5 - 2} \Rightarrow \frac{\sqrt{5} - \sqrt{2}}{3}$ Ans

Exercise 1.6

Laws of exponents

1. Find:-

i) $64^{\frac{1}{2}} \Rightarrow (2 \times 2 \times 2 \times 2 \times 2 \times 2)^{\frac{1}{2}} = (2^6)^{\frac{1}{2}} = 2^{\frac{6 \times 1}{2}} = 2^3$

ii) $32^{\frac{1}{5}} \Rightarrow (2 \times 2 \times 2 \times 2 \times 2)^{\frac{1}{5}} = (2^5)^{\frac{1}{5}} = 2^{\frac{5 \times 1}{5}} = 2^1 = 2$ Ans

iii) $125^{\frac{1}{3}} \Rightarrow (5 \times 5 \times 5)^{\frac{1}{3}} = (5^3)^{\frac{1}{3}} = 5^{\frac{3 \times 1}{3}} = 5$ Ans

2. Find:

i) $9^{\frac{3}{2}} = (3 \times 3)^{\frac{3}{2}} = (3^2)^{\frac{3}{2}} = 3^{2 \times \frac{3}{2}} = 3^3 = 3 \times 3 \times 3 = 27$ Ans

ii) $32^{\frac{2}{5}} = (2 \times 2 \times 2 \times 2 \times 2)^{\frac{2}{5}} = (2^5)^{\frac{2}{5}} = 2^{\frac{5 \times 2}{5}} = 2^2 = 2 \times 2 = 4$ Ans

iii) $16^{\frac{3}{4}} = (2 \times 2 \times 2 \times 2)^{\frac{3}{4}} = (2^4)^{\frac{3}{4}} = 2^{\frac{4 \times 3}{4}} = 2^3 = 2 \times 2 \times 2 = 8$ Ans

iv) $125^{\frac{-1}{3}} = (5 \times 5 \times 5)^{\frac{-1}{3}} = (5^3)^{\frac{-1}{3}} = 5^{\frac{3 \times (-1)}{3}} = 5^{-1} = \frac{1}{5}$ Ans

Q2. Simplify:

i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{3}} = 2^{\frac{2}{3} + \frac{1}{3}} = 2^{\frac{2+1}{3}} = 2^{\frac{3}{3}} = 2^1 = 2$ Ans

ii) $\left[\frac{1}{3^3}\right]^7 = \frac{1}{3^{3 \times 7}} = \frac{1}{3^{21}} = 3^{-21}$ Ans

iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{3}{4}}} = 11^{\frac{1}{2}} \times 11^{-\frac{3}{4}} = 11^{\frac{1}{2} + (-\frac{3}{4})} = 11^{\frac{2}{4} - \frac{3}{4}} = 11^{-\frac{1}{4}} = \frac{1}{11^{\frac{1}{4}}}$ Ans

iv) $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}} = (7 \times 8)^{\frac{1}{2}} = 56^{\frac{1}{2}}$ Ans