Chapter ⇒ 2 Polynomials

Ex ⇒ 2.1

1 Which of the following expressions are polynomials in one variable and which are not ? State reason for your answer.

i) $4x^2 - 3x$

⇒ Yes, this expression is a polynomial is one Variable (x).

ii) $y^2 + \sqrt{2}$

⇒ Yes, this expressions is a polynomial is one Variable (y)

iii) $3\sqrt{t} + t\sqrt{2}$

⇒ No, it can be observed that the exponents of variable (t) in term $3\sqrt{t}$ is $\frac{1}{2}$ which is not a whole number Therefore this expressions is not a polynomial.

iv) $x^{10} + y^3 + t^{50}$

⇒ No, It can be observed that this expressions is a polynomial in three variables x, y and therefore, This expression is not a polynomial in one variable but is polynomial in 3 variables

Write the coefficients of x^2 in each of the following.

$2 + x^2 + x$

The coefficient in $2+x^2+x$ of x^2 is 1.

$2-x^2+x^3$

The coefficient $2-x^2+x^3$ of x^2 is -1.

$\frac{\pi}{2}x^2+x$

The coefficient in $\frac{\pi}{2}x^2+x$ of x^2 is $\frac{\pi}{2}$.

$\sqrt{2}x-1$

The coefficient in $\sqrt{2}x-1$ of x^2 is 0.

Give one examples each of the following binomial of degree 35, and of a monomial of degree 100.

Binomial of degree $35 = x^{35}+30y$

Monomial of degree $100 = 11x^{100}$

Write the degree of each of the following polynomials:-

$5x^3+4x^2+7x$

Degree of Polynomial = 3

$4-y^2$

Degree of Polynomial = 2.

$5t-\sqrt{7}$

Degree of Polynomial = 1

3

Degree of Polynomial = 0

Q5. Classify the following as linear, quadratic and cubic polynomials:

i) $x^2 + x$ ⇒ Quadratic Polynomial.

ii) $x - x^3$ ⇒ Cubic polynomial.

iii) ~~$7x^3$~~ ⇒ $y + y^2 + 4$ ⇒ Quadratic polynomial.

iv) $1 + x$ ⇒ Linear Polynomial.

v) $3t$ ⇒ Linear Polynomial.

vi) r^2 ⇒ Quadratic polynomial.

vii) $7r^3$ ⇒ Cubic polynomial.

viii)

Exercise 2.2

Q 1. Find the value of the Polynomial $5x - 4x^2 + 3$ at

i) $x = 0$

Solve $\Rightarrow 5x - 4x^2 + 3$

$\Rightarrow 5(0) - 4(0)^2 + 3$

$\Rightarrow 0 - 0 + 3$

$\Rightarrow 3$ Ans

ii) $x = -1$

Solve $\Rightarrow 5x - 4x^2 + 3$

$\Rightarrow 5(-1) - 4(-1)^2 + (3)$

$\Rightarrow -5 - 4(1) + 3$

$\Rightarrow -5 - 4 + 3$

$\Rightarrow -9 + 3$

$\Rightarrow -6$ Ans

iii) $x = 2$

Solve $\Rightarrow 5x - 4x^2 + 3$

$\Rightarrow 5(2) - 4(2)^2 + 3$

$\Rightarrow 10 - 4(4) + 3$

$\Rightarrow 13 - 16$

$\Rightarrow -3$ Ans

Q 2. Find $P(0)$, $P(1)$ and $P(2)$ for each of the following polynomials:

i) $P(y) = y^2 - y + 1$

$\Rightarrow P(0) = 0^2 - 0 + 1$

$\Rightarrow 0 - 0 + 1$

$\Rightarrow 1$ Ans

$\Rightarrow P(1) = 1^2 - 1 + 1$

$\Rightarrow 1 - 1 + 1$

$\Rightarrow 1$ Ans

$\Rightarrow P(2) = 2^2 - 2 + 1$

$\Rightarrow 4 - 2 + 1$

$\Rightarrow 2 + 1$

$\Rightarrow 3$ Ans

ii) $P(t) = 2 + t + 2t^2 - t^3$

Solve $\Rightarrow P(0) = 2 + 0 + 2(0)^2 - 0^3$

$\Rightarrow 2$ Ans

$\star P(1) = 2 + 1 + 2(1)^2 - 1^3$

$\Rightarrow 3 + 2(1) - 1$

$\Rightarrow 3 + 2 - 1$

$\Rightarrow 4$ Ans

$\star P(2) = 2 + 2 + 2(2)^2 - 2^3$

$\Rightarrow 4 + 2(4) - 8$

$\Rightarrow 4 + 8 - 8$

$\Rightarrow 4$ Ans

iii) $P(x) = x^3$

Solve $P(0) = 0^3$

$= 0$

$P(1) = 1^3$

$\Rightarrow 1$

$P(2) = 2^3$

$\Rightarrow 2 \times 2 \times 2$

$\Rightarrow 8$ Ans

iv) $p(x) = (x-1)(x+1)$

Solve $\Rightarrow P(0) = (0-1)(0+1)$

$\Rightarrow (-1) \cdot (1)$

$\Rightarrow -1$ Ans

$P(1) = (1-1)(1+1)$

$\Rightarrow (0)(2)$

$\Rightarrow 0$ Ans

$P(2) = (2-1)(2+1)$

$\Rightarrow (1)(3)$

$\Rightarrow 3$ Ans

Q3. Verify whether the following are zeroes of the polynomial, indicated against them

i) $P(x) = 3x + 1, \; x = -\frac{1}{3}$

Solve $\Rightarrow P\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) + 1$

$\Rightarrow -1 + 1$

$\Rightarrow 1 - 1 = 0$

$\therefore -\frac{1}{3}$ is zero of $P(x)$

ii) $P(x) = 5x - \pi, \; x = \frac{4}{5}$

Solve $P\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - \pi$

$4 - \pi$

$4 - \pi \neq 0$

$\therefore \frac{4}{5}$ is not zero of $p(x)$.

iii) $P(x) = x^2 - 1, \; x = 1, -1$

Solve $\Rightarrow P(1) = 1^2 - 1$

$\Rightarrow 1 - 1 = 0$

$P(-1) = (-1)^2 - 1$

$= 1 - 1$

$= 0$

$\therefore -1, 1$ both are zeroes of $P(x)$

iv) $P(x) = (x+1)(x-2), \; x = -1, 2$

Solve $\Rightarrow P(-1) = (-1+1)(-1-2)$

$\Rightarrow (1-1) \cdot (-3)$

$\Rightarrow (0)(-3) = 0$

$\Rightarrow P(2) = (2+1) \cdot (2-2)$

$\Rightarrow (3)(0)$

$\Rightarrow 0$

$\therefore 1, 2$ both are zeroes of $P(x)$

v) $P(x) = x^3, \; x = 0$.

Solve $\Rightarrow P(0) = 0^3$

$= 0$

$\therefore 0$ is zero of $P(x)$

vii) $P(x) = lx + m$; $x = \frac{-m}{l}$

$\Rightarrow P\left(\frac{-m}{l}\right) = l\left(\frac{-m}{l}\right) + m$

$\Rightarrow -m + m$

$\Rightarrow m - m$

$= 0$

$\therefore \frac{-m}{l}$ is zero of $P(x)$

viii) $P(x) = 3x^2 - 1$, $x = \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Solve $\Rightarrow P\left(\frac{-1}{\sqrt{3}}\right) = 3\left(\frac{-1}{\sqrt{3}}\right)^2 - 1$

$\Rightarrow 3\left(\frac{1}{3}\right) - 1$

$= 0$ Ans

$P\left(\frac{1}{\sqrt{3}}\right) = 3\left(\frac{1}{\sqrt{3}}\right)^2 - 1$

$\Rightarrow 3\left(\frac{1}{3}\right) - 1$

$\Rightarrow 0$

$\therefore \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ are zeroes $p(x)$

4. Find the zero of the polynomial in each of the following cases:-

i) $P(x) = x + 5$

Solve $\Rightarrow x + 5 = 0$

$x = -5$

ii) $P(x) = x - 5$

Solve $x - 5 = 0$

$\Rightarrow x = 5$

iii) $P(x) = 2x + 5$

Solve $\Rightarrow 2x + 5 = 0$

$2x = -5$

$x = \frac{-5}{2}$

vi) $P(x) = 3x - 2$

Solve $\Rightarrow 3x - 2 = 0$

$3x = 2$

$x = \frac{2}{3}$

viii) $P(x) = 2x + 1$, $x = \frac{1}{2}$

Sol $\Rightarrow P\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1$

$\Rightarrow 1 + 1 = 2$

$\therefore \frac{1}{2}$ is not a zero of $P(x)$

vi) $P(x) = 3x$

Solve $3x = 0$

$x = \frac{0}{3}$

$x = 0$

vii) $P(x) = ax, a \neq 0$

Solve $ax = 0$

$x = \frac{0}{a}$

$x = 0$. Ans

viii) $P(x) = cx + d, c \neq 0$; c, d are real numbers;

Solve $\Rightarrow cx + d = 0$

$cx = -d$

$x = \frac{-d}{c}$

Ans

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Exercise 2.3

1. Find the remainder when x^3+3x^2+3x+1 is divided by.

(i) $x+1$

$\Rightarrow x+1=0$

$x=-1$

$\Rightarrow x^3+3x^2+3x+1$

$\Rightarrow (-1)^3+3(-1)^2+3(-1)+1$

$\Rightarrow -1+3(1)-3+1$

$\Rightarrow -1+3-3+1$

$\Rightarrow 0$ Ans

(ii) x

Solve $\Rightarrow x=0$

$\Rightarrow x^3+3x^2+3x+1$

$\Rightarrow 0^3+3(0)^2+3(0)+1$

$\Rightarrow 0+0+0+1$

$\Rightarrow 1$ Ans

(iii) $(5+2x)$

$5+2x=0$

$\Rightarrow x=\frac{-5}{2}$

$\Rightarrow x^3+3x^2+3x+1$

$\Rightarrow \left(\frac{-5}{2}\right)^3+3\left(\frac{-5}{2}\right)^2+3\left(\frac{-5}{2}\right)+1$

$\Rightarrow \frac{-125}{8}+3\left(\frac{25}{4}\right)+3\left(\frac{-5}{2}\right)+1$

$\Rightarrow \frac{-125}{8}+\frac{75}{4}-\frac{15}{2}+1 \Rightarrow \frac{-125+150-60+8}{8}$

(iv) $x-\frac{1}{2}$

Solve $\Rightarrow x-\frac{1}{2}=0$

$\Rightarrow x=\frac{1}{2}$ $\Rightarrow x^3+3x^2+3x+1$

$\Rightarrow \left(\frac{1}{2}\right)^3+3\left(\frac{1}{2}\right)^2+3\left(\frac{1}{2}\right)+1$

$\Rightarrow \frac{1}{8}+3\left(\frac{1}{4}\right)+\frac{3}{2}+1$

$\Rightarrow \frac{1}{8}+\frac{3}{4}+\frac{3}{2}+\frac{1}{1} \Rightarrow \frac{1+6+12+8}{8}$

$\Rightarrow \frac{27}{8}$ Ans

(v) $x+\pi$

Solve $\Rightarrow x+\pi=0$

$x=-\pi$

$\Rightarrow x^3+3x^2+3x+1$

$\Rightarrow \pi^3+3(-\pi)^2+3(-\pi)+1$

$\Rightarrow -\pi^3+3\pi^2-3\pi+1$ Ans

$\Rightarrow \frac{158 - 185}{8}$

$\Rightarrow \frac{-27}{8}$

Ans

Q2. Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Solve $\Rightarrow$ $x - a = 0$

$x = a$

$\Rightarrow x^3 - ax^2 + 6x - a$

$\Rightarrow a^3 - a(a)^2 + 6(a) - a$

$\Rightarrow a^3 - a^3 + 6a - a$

$\Rightarrow 0 + 5a$

$\Rightarrow 5a$ Ans

Q3. Check whether $7 + 3x$ is the factor of $P(x) = 3x^3 + 7x$.

Sol. $7 + 3x = 0$

$x = \frac{-7}{3}$

$P(x) = 3x^3 + 7x$

$\Rightarrow P\left(\frac{-7}{3}\right) = 3\left(\frac{-7}{3}\right)^3 + 7\left(\frac{-7}{3}\right)$

$-7 \times 7 \times 7 = -343$

$\Rightarrow \frac{(-343)}{9} \cdot \frac{-49}{3}$

$49 \times 3 = 147$

$\Rightarrow \frac{-343}{9} - \frac{49}{3}$ $\Rightarrow \frac{-343 - 147}{9} = -490$ Ans

$7 + 3x$ is not a factor of $P(x)$ as remainder is not equal to zero.

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Exercise 2.4

1. Determine which of the following polynomials has $(x+1)$ a factor

$(x+1) = x+1=0 \Rightarrow x=-1$

(i) x^3+x^2+x+1

Solve $\Rightarrow x+1=0$

$\Rightarrow x=-1$

$\Rightarrow x^3+x^2+x+1$

$\Rightarrow (-1)^3+(-1)^2+(-1)+1$

$\Rightarrow -1+1-1+1$

$\Rightarrow 2-2$

$\Rightarrow 0$ Ans

$x+1$ is factor of given Polynomials

(ii) $x^4+x^3+x^2+x+1$

Solve $\Rightarrow x+1=0$

$x=-1$

$\Rightarrow x^4+x^3+x^2+x+1$

$\Rightarrow (-1)^4+(-1)^3+(-1)^2+(-1)+1$

$\Rightarrow 1-1+1-1+1$

$\Rightarrow 3-2=1$

$\Rightarrow 1$ Ans

$x+1$ is not factor of given Polyno

(iii) $x^4+3x^3+3x^2+x+1$

Solve: $(-1)^4+3(-1)^3+3(-1)^2+(-1)+1$

$\Rightarrow 1+3(-1)+3(1)-1+1$

$\Rightarrow 1-3+3-1+1$

$\Rightarrow 5-4$

$\Rightarrow 1$

$x+1$ is not factor of given polynomials.

(iv) $x^3-x^2-(2+\sqrt{2})x+\sqrt{2}$

Solve $\Rightarrow (-1)^3-(-1)^2-2(-1)-\sqrt{2}(-1)$

$\Rightarrow -1-1+2+\sqrt{2}+\sqrt{2}$

$\Rightarrow 2-2+2\sqrt{2}$

$\Rightarrow 0+2\sqrt{2}$

$\Rightarrow 2\sqrt{2}$

$x+1$ is not factor of the given Polynomials.

Q2. Use the Factor theorem to determine whether $g(x)$ is a facto of $P(x)$ in each of the following cases.

i) $P(x) = 2x^3+x^2-2x-1$; $g(x) = x+1$.

$\Rightarrow x+1=0$

$x=-1$

$P(x) = 2x^3+x^2-2x-1$.

$P(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$\Rightarrow 2(-1) + 1 + 2 - 1$

$\Rightarrow -2 + 1 + 2 - 1$

$\Rightarrow 3 = 3$

$\Rightarrow 0$ Ans $g(x)$ is factor of $P(x)$.

(ii) $P(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$

Sol $\Rightarrow$ $x + 2 = 0$

$x = -2$

$P(x) = x^3 + 3x^2 + 3x + 1$

$\Rightarrow P(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

$\Rightarrow -8 + 3(4) - 6 + 1$

$\Rightarrow -8 + 12 - 6 + 1$

$\Rightarrow 13 - 14$

$\Rightarrow -1$ Ans $g(x)$ is not factor of $P(x)$.

(iii) $P(x) = x^2 - 4x^2 + x + 6$, $g(x) = x - 3$.

Solve $\Rightarrow$ $x - 3 = 0$.

$x = 3$

$P(x) = x^3 - 4x^2 + x + 6$

$P(3) = (3)^3 - 4(3)^2 + (3) + 6$

$\Rightarrow 27 - 4(9) + 9$

$\Rightarrow 36 - 36$

$\Rightarrow 0$ Ans So, $g(x)$ is a factor of $P(x)$.

Q3. Find the value of k, if $x - 1$ is a factor of $P(x)$ in each of the following cases:

(i) $P(x) = x^2 + x + k$

Sol $\Rightarrow$ $x - 1 = 0$

$x = 1$

$p(x) = x^2 + x + K$

$\Rightarrow 1^2 + 1 + K = 0$

$\Rightarrow 1 + 1 + K = 0$

$\Rightarrow 2 + K = 0$

$\Rightarrow \boxed{K = -2}$

ii) $P(x) = 2x^2 + kx + \sqrt{2}$

Solve $\Rightarrow P(x) = 2(1)^2 + k(1) + \sqrt{2} = 0$

$\Rightarrow 2(1) + k + \sqrt{2} = 0$

$2 + k + \sqrt{2} = 0$

$\boxed{k = -2 - \sqrt{2}}$

iii) $P(x) = kx^2 - \sqrt{2}x + 1$

Solve $\Rightarrow x - 1 = 0$

$x = 1$

$P(x) = kx^2 - \sqrt{2}x + 1$

$k(1)^2 - \sqrt{2}(1) + 1 = 0$

$k(1) - \sqrt{2} + 1 = 0$

$\boxed{k = \sqrt{2} - 1}$ Ans.

iv) $P(x) = kx^2 - 3x + k$

Sol $\Rightarrow x - 1 = 0$

$x = 1$

$P(x) = kx^2 - 3x + k$

$k(1)^2 - 3(1) + k = 0$

$k(1) - 3 + k = 0$

$k - 3 + k = 0$

$2k = 3$

$\boxed{k = \frac{3}{2}}$ Ans.

Q4: Factorise :

(i) $12x^2 - 7x + 1$

Solve $\Rightarrow 12x^2 - 4x - 3x + 1$

$\Rightarrow 4x(3x - 1) - 1(3x - 1)$

$\Rightarrow (4x - 1)(3x - 1)$ Ans.

(ii) $2x^2 + 7x + 3$

Solve $\Rightarrow 2x^2 + 6x + x + 3$

$\Rightarrow 2x(x + 3) + 1(x + 3)$

$\Rightarrow (2x + 1)(x + 3)$ Ans.

2 | 6
3 | 3
1

$6 + 1 = 7$

i) $6x^2 + 5x - 6$

lve $\Rightarrow 6x^2 + 9x - 4x - 6$

$\Rightarrow 3x(2x + 3) - 2(2x + 3)$

$\Rightarrow (3x - 2)(2x + 3)$ Ans.

(iv) $3x^2 - x - 4$

Solve $\Rightarrow 3x^2 + 3x - 4x - 4$

$\Rightarrow 3x(x + 1) - 4(x + 1)$

$\Rightarrow (3x - 4)(x + 1)$

$6 \times 6 = 36$

4 (2 | 36
2 | 18
3 | 3
3 | 3
1

9 (3 | 3
3 | 3
1

$9 - 4 = 5$

$3 - 4 = -1$

: Factorise

$x^3 - 2x^2 - x + 2$

$x - 1 = 0, \; x = 1$

e $\Rightarrow$ Let $x - 1$ is factor of polynomial

$x = 1$

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$\Rightarrow x^3 - 2x^2 - x + 2$

$\Rightarrow 1^3 - 2(1)^2 - 1 + 2$

$\Rightarrow 1 - 2(1) - 1 + 2$

$\Rightarrow 3 - 3 = 0$

$x-1 \overline{)\, x^3 - 2x^2 - x + 2}$ $\quad x^2 - x$

$x^3 - x^2$

$-x^2 - x + 2$

$-x^2 + x$

$-2x + 2$

$-2x + 2$

0

$= x^2 - x - 2$

$= x^2 + x - 2x - 2$

$\Rightarrow x(x+1) - 2(x+1)$

$\Rightarrow (x-2)\cdot(x+1)\cdot(x-1)$ Ans

iii) $x^3 - 3x^2 - 9x - 5$

Solve: Let $x+1$ is factor of polynomial

$x + 1 = 0$

$x = -1$

$\Rightarrow x^3 - 3x^2 - 9x - 5$

$\Rightarrow (-1)^3 - 3(-1)^2 - 9(-1) - 5$

$\Rightarrow -1 - 3(1) + 9 - 5$

$\Rightarrow -1 - 3 + 9 - 5$

$\Rightarrow 9 - 9$

$\Rightarrow 0$ Ans

$x+1 \overline{)\, x^3 - 3x^2 - 9x - 5}$ $\quad x^2 - 4x - 5$

$x^3 + x^2$

$-3x^2 - 2^2 = -4x^2$

$-4x^2 - 4x$

$-9x - 5$

$-5x - 5$

$-5x - 5$

0

$4x - 9x = -5x$

$\Rightarrow x^2 - 4x - 5$

$x^2 + x - 5x - 5$

$\Rightarrow x(x+1) - 5(x+1)$

$\Rightarrow (x-5)(x+1)\cdot(x+1)$ Ans

$(x+1)$

5

$1, 5$

iii) $x^3 + 13x^2 + 32x + 20$

Solve: Let $x+1$ is factor of polynomial

$x + 1 = 0$

$x = -1$

$\Rightarrow x^3 + 13x^2 + 32x + 20$

$41x + 1$

$= (-1)^3 + 13(-1)^2 + 32(-1) + 20$

$\Rightarrow -1 + 13(1) - 32 + 20$

$\Rightarrow -1 + 13 - 32 + 20$

$\Rightarrow 33 - 33$

$\Rightarrow 0$ Ans

$x+1 \;\big)\; x^3 + 13x^2 + 32x + 20 \;\big(\; x^2 + 12x + 20$

$x^3 + x^2$

$12x^2 + 32x + 20$

$12x^2 + 12x$

$20x + 20$

$20x + 20$

0

$\Rightarrow x^2 + 12x + 20$

$\Rightarrow x^2 + 2x + 10x + 20$

$\Rightarrow x(x+2) + 10(x+20)$

$\Rightarrow (x+10)(x+2)(x+1)$ Ans

$(x+1)$

2	20
12	10

$2 + 10 = 12$

iv) $2y^3 + y^2 - 2y - 1$

$\Rightarrow y^2(2y+1) - 1(2y+1)$

$(y^2 - 1^2) \cdot (2y+1)$

$(y+1)(y-1)(2y+1)$ Ans

$i^2 = i \times i = -1$

$(a^2 - b^2) = (a+b)(a-b)$

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Q. Use Suitable Identities to find the following products:

Exercise 2.5

(i) $(x+4)\cdot(x+10)$

Solve ⇒ $A=4$ and $B=10$

$(x+4)\cdot(x+10)$

$x^2+ax+bx+ab$

$\Rightarrow x^2+x(4+10)+4\times10$

$\rightarrow x^2+14x+40x$ Ans

(ii) $(x+8)(x-10)$

Solve ⇒ $A=8$ and $B=10$

$x^2+Ax-Bx-AB$

$x^2+x(8+(-10)+8\times(-10)$

$\Rightarrow x^2+2x-80$ Ans

(iii) $(3x+4)(3x-5)$

Solve ⇒ $(x+a)(x+b)=x^2+(a+b)x+ab$

$(3x)^2+[4+(-5)]3x+4\cdot(-5)$

$9x^2+(4-5)3x-20$

$9x^2-1(3x)-20$

$9x^2-3x-20$ Ans

(iv) $\left(y^2+\frac{3}{2}\right)\cdot\left(y^2-\frac{3}{2}\right)$

Solve ⇒ $(a+b)(a-b)=a^2-b^2$

$\Rightarrow (y^2)^2-\left(\frac{3}{2}\right)^2$

$\Rightarrow y^4-\frac{9}{4}$

Ans

(v) $(3-2x)(3+2x)$

Solve ⇒ $(a-b)(a+b)=a^2-b^2$

$\Rightarrow 3^2-(2x)^2$

$\Rightarrow 9-4x^2$ Ans

Q4 Evaluate the following products without multiply directly.

(i) 103×107

Solve $(100+3)(100+7)$

$(x+a)(x+b) = x^2 + (a+b)x + ab$

$\Rightarrow 100^2 + (3+7)100 + 3 \times 7$

$\Rightarrow 10000 + 10 \times 100 + 21$

$\Rightarrow 10000 + 1000 + 21$

$\Rightarrow$ ~~11021~~ Ans

(ii) 95×96

Solve $(100-5)(100-4)$

$\Rightarrow (x+a)(x+b) = x^2 + (a+b)x + ab$

$\Rightarrow 100^2 + (-5-4)100 + (-4)(-5)$

$\Rightarrow 10,000 + (-9)100 + 20$

$\Rightarrow 10,000 - 900 + 20$ $\Rightarrow 10020 - 900$

$\Rightarrow 9120$ Ans

(iii) 104×96

$(100+4) \times (100-4)$

$(a+b)(a-b) = a^2 - b^2$

$\Rightarrow 100^2 - 4^2$

$\Rightarrow 10,000 - 16$

$\Rightarrow 9984$ Ans

Q3 Factorise the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$

Solve $\Rightarrow (3x)^2 + 2(3x)(y) + y^2$ $\Rightarrow a^2 + 2ab + b^2 \Rightarrow (a+b)^2$

$\Rightarrow (3x+y)^2$ Ans

(ii) $4y^2 - 4y + 1$ $A^2 - 2ab + b^2 \Rightarrow (a-b)^2$

Solve $\Rightarrow (2y)^2 - 2(2y)(1) + 1^2$

$\Rightarrow (2y-1)^2$

(iii) $y^2 - \frac{y^2}{100}$

Solve $\Rightarrow y^2 - \left(\frac{y}{10}\right)^2$ $\Rightarrow (a^2 - b^2) = (a+b)(a-b)$

$\Rightarrow \left(y + \frac{y}{10}\right)\left(y - \frac{y}{10}\right)$ Ans

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4. Expand each of the following, using suitable identities :-

(i) $(x+2y+4z)^2$

Solve ⇒ $(a+b+c)^2 \Rightarrow a^2+b^2+c^2+2ab+2bc+2ac$

⇒ $x^2+(2y)^2+(4z)^2+2(x)(2y)+2(2y)(4z)+2(x)(4z)$

$x^2+4y^2+16z^2+4xy+16yz+8xz$. Ans

(ii) $(2x-y+z)^2$

Solve ⇒ $(a+b+c)^2 \Rightarrow a^2+b^2+c^2+2ab+2bc+2ac$

⇒ $(2x)^2+(-y)^2+z^2+2(2x)(-y)+2(-y)(z)+2(2x)\cdot(z)$

⇒ $4x^2+y^2+z^2-4xy-2yz+4xz$ Ans

(iii) $(-2x+3y+2z)^2$

Solve ⇒ $a^2+b^2+c^2+2ab+2bc+2ca$

⇒ $(-2x)^2+(3y)^2+(2z)^2+2(-2x)(3y)+2(3y)(2z)+2(2z)(-2x)$

⇒ $4x^2+9y^2+4z^2-12xy+12yz-8zx$. Ans

(iv) $(3a-7b-c)^2$

Solve ⇒ $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$.

⇒ $(3a)^2+(-7b)^2+(-c)^2+2(3a)(-7b)+2(-7b)(-c)+2(-c)(3a)$

⇒ $9a^2+49b^2+c^2-42ab+14bc-6ca$. Ans

(v) $(-2x+5y-3z)^2$

Solve ⇒ $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$.

⇒ $(-2x)^2+(5y)^2+(-3z)^2+2(-2x)(5y)+2(5y)(-3z)+2(-3z)$

⇒ $4x^2+25y^2+9z^2-20xy-30yz+12zx$. Ans

(vi) $\left[\frac{1}{4}a-\frac{1}{2}b+1\right] \Rightarrow \left(\frac{1}{4}a\right)^2+\left(-\frac{1}{2}b\right)^2+1^2+2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right)+2\left(-\frac{1}{2}b\right)$

Solve ⇒ $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$ ← $2(1)\left(\frac{1}{2}a\right)$

⇒ $\frac{1}{16}a^2+\frac{1}{4}b^2+1-\frac{1}{4}ab-b+\frac{1}{2}a$ Ans

5 Factorise :-

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24xz - 16xz$

Solve ⇒ $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2$

⇒ $(2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$

⇒ $(2x + 3y - 4z)^2$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Solve ⇒ $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2$

$(-\sqrt{2}x)^2 + (y)^2 + (\sqrt{8}z)^2 + 2(-\sqrt{2}x)(y) + 2(y)(\sqrt{8}z) + 2(-\sqrt{2}x)$

⇒ $(-\sqrt{2}x)^2 + y^2 + (2\sqrt{2}z)^2 + 2(-\sqrt{2}x)(y) + 2(y)(2\sqrt{2}z) + 2(2\sqrt{2}z)(-\sqrt{2}x)$

⇒ $(-\sqrt{2}x + y + 2\sqrt{2}z)^2$ Ans

6. Write the following cubes in expanded form

(i) $(2x+1)^3$

Solve ⇒ $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

⇒ $(2x)^3 + 1^3 + 3(2x)^2 1 + 3(2x)(1)^2$

⇒ $8x^3 + 1 + 3(4x^2 + 6x(1)$

⇒ $8x^3 + 1 + 12x^2 + 6x$

⇒ $8x^3 + 12x^2 + 6x + 1$ Ans

(ii) $(2a - 3b)^3$

Solve ⇒ $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

⇒ $(2a)^3 + (-3b)^3 + 3(2a)^2(-3b) + 3(2a)(-3b)^2$

⇒ $8a^3 - 27b^3 - 9b(4a^2) + 6a(9b^2)$

⇒ $8a^3 - 27b^3 - 36a^2b + 54ab^2$ Ans

(iii) $\left[\frac{3}{2}x + 1\right]^3$

Solve ⇒ $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

⇒ $\left(\frac{3}{2}x\right)^3 + 1^3 + 3\left(\frac{3}{2}x\right)^2 1 + 3\left(\frac{3}{2}x\right)1^2$

$\Rightarrow \frac{21}{8}x^3 + 1 + 3\left(\frac{9}{4}x^2\right) + \frac{9}{2}x(1)$

$\Rightarrow \frac{27}{8}x^3 + 1 + \frac{27}{4}x^2 + \frac{9}{2}x$

$\Rightarrow \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$ Ans

(iv) $\left[x - \frac{2}{3}y\right]^3$

Solve $\Rightarrow (a+b)^3 \Rightarrow a^3 + b^3 + 3a^2b + 3ab^2$

$\Rightarrow x^3 + \left(-\frac{2}{3}y\right)^3 + 3(x)^2\left(-\frac{2}{3}y\right) + 3x\left(-\frac{2}{3}y\right)^2$

$\Rightarrow x^3 - \frac{8}{27}y^3 + x^2(-2y) + \frac{3x}{3}\left(\frac{4}{9}y^2\right)$

$\Rightarrow x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$ Ans

7. Evaluate the following using suitable identities.

(i) $(99)^3$

Solve $\Rightarrow (100-1)^3$

$\Rightarrow (a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$\Rightarrow (100)^3 + (-1)^3 + 3(100)^2(-1) + 3(100)(-1)^2$

$\Rightarrow 1000000 - 1 - 3(10000) + 300(1)$

$\Rightarrow 1000000 - 1 - 30000 + 300$

$\Rightarrow 1000000 - 30001$

$\Rightarrow 970299$ Ans

$1,000,000$
$-\ 30,001$
970299 Ans

(ii) $(102)^3$

Solve $(100+2)$

$\Rightarrow (a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$\Rightarrow 100^3 + 2^3 + 3(100)^2(2) + 3(100)(2)^2$

$\Rightarrow 1000000 + 8 + 6(10000) + 300(4)$

$\Rightarrow 1000000 + 8 + 60000 + 1200$

⇒ 1061208 Ans

(ii) $(998)^3$

Solve ⇒ $(1000-2)^3$

⇒ $(a+b)^3 = a^3+b^3+3a^2b+3ab^2$

⇒ $1000^3 + (-2)^3 + 3(1000)^2(-2) + 3(1000)(-2)^2$

⇒ $1,000,000,000 - 8 - 6(1,000,000) + 12000$

⇒ $1,000,000,000 - 8 - 6,000,000 + 12000$

⇒ $1,000,012,000 - 6,000,008$

⇒ 994011992 Ans

Q8 Factorise each of the following:-

(i) $8a^3+b^3+12a^2b+6ab^2$

Solve ⇒ $a^3+b^3+3a^2b+3ab^2 = (a+b)^3$

⇒ $(2a)^3+(b)^3+3(2a)^2b+3(2a)(b)^2$

⇒ $(2a+b)^3$ Ans.

$(2a+b)$ $(2a+b)$ $(2a+b)$ Ans

(ii) $8a^3-b^3-12a^2b+6ab^2$

Solve ⇒ $a^3+b^3+3a^2b+3ab^2 = (a+b)^3$

⇒ $(2a)^3+(-b)^3+3(2a)^2(-b)+3(2a)(-b)^2$

⇒ $[2a+(-b)]^3$

⇒ $(2a-b)^3$ Ans

$(2a-b)$. $(2a-b)$ $(2a-b)$

(iii) $27-125a^3-135a+225a^2$

Solve ⇒ $a^3+b^3+3a^2b+3ab^2 = (a+b)^3$

⇒ $(3)^3+(-5a)^3+3(3)^2(-5a)+3(3)(-5a)^2$

⇒ $[3+(-5a)]^3$

⇒ $(3-5a)^3$

⇒ $(3-5a)$, $(3-5a)$ $(3-5a)$ Ans

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

Solve ⇒ $a^3 + b^3 + 3a^2b + 3ab^2 = (a+b)^3$

⇒ $(3)^3 + (-5a)^3 + 3(3)^2(-5a) + 3(3)(-5a)^2$

⇒ $[3 + (-5a)]^3$

⇒ $(3-5a)^3$ Ans

⇒ $(4a)^3 + (-3b)^3 + 3(4a)^2(-3b) + 3(4a)(-3b)^2$

⇒ $[4a + (-3b)]^3$

⇒ $[4a - 3b]^3$ Ans

⇒ $(4a-3b)\ (4a-3b)\ (4a-3b)$ Ans

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Solve ⇒ $a^3 + b^3 + 3a^2b + 3ab^2 = (a+b)^3$

⇒ $(3p)^3 + \left(-\frac{1}{6}\right)^3 + 3(3p)^2\left(-\frac{1}{6}\right) + 3(3p)\left(-\frac{1}{6}\right)^2$

⇒ $\left[3p + \left(-\frac{1}{6}\right)\right]^3$

⇒ $\left(3p + \frac{1}{6}\right)^3$

⇒ $\left(3p+\frac{1}{6}\right)\left(3p+\frac{1}{6}\right)\left(3p+\frac{1}{6}\right)$ Ans

Q9. Verify: (i) $x^3 + y^2 = (x+y)(x^2 - xy + y^2)$

Solve ⇒ By RHS

⇒ $x\cdot(x^2 - xy + y^2) + y\cdot(x^2 - xy + y^2)$

⇒ $x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$

⇒ $x^3 + y^3$

LHS = RHS

(ii) $x^3 - y^3 = (x-y)\cdot(x^2 + xy + y^2)$

Solve By R.H.S

$x(x^2+xy+y^2) - y(x^2+xy+y^2)$

$\Rightarrow x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3$

$\Rightarrow x^3 - y^3$

L.H.S = R.H.S

10. Factorise each of the following :-

(i) $27y^3 + 125z^3$

Solve $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

$\Rightarrow (3y)^3 + (5z)^3$

$\Rightarrow (3y+5z)[(3y)^2 - (3y)(5z) + (5z)^2]$

$\Rightarrow (3y+5z)[9y^2 - 15yz + 25z^2]$ Ans

(ii) $64m^3 - 343n^3$

Solve $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

$\Rightarrow (4m)^3 - (7n)^3$

$\Rightarrow (4m-7n)[(4m)^2 + (4m)(7n) + (7n)^2]$

$\Rightarrow (4m-7n)[16m^2 + 28mn + 49n^2]$ Ans

11. Factorise :-

(i) $27x^3 + y^3 + z^3 - 9xyz$

$\Rightarrow (3x)^3 + y^3 + z^3 - 3(3x)yz$

$a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc + ac)$

$\Rightarrow (3x+y+z)[(3x)^2 + y^2 + z^2 - (3x)y - yz - (3x)z]$

$\Rightarrow (3x+y+z)[9x^2 + y^2 + z^2 - 3xy - yz - 3xz]$

12. Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x+y+z)[(x-y)^2 + (y-z)^2 + (z-x)^2]$

$(a-b)^2 = a^2 + b^2 - 2ab$

By RHS $= \frac{1}{2}(x+y+z)[x^2 + y^2 + 2xy + y^2 + z^2 - 2yz + z^2 + x^2 - 2zx]$

$\Rightarrow \frac{1}{2}(x+y+z)[2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx]$

$\Rightarrow \frac{1}{2} \times 2 (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$\Rightarrow (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$\Rightarrow x^3+y^3+z^3-3xyz$

$x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

LHS = RHS

Q13 If $x+y+z=0$, Show that $x^3+y^3+z^3=3xyz$

Solve $\Rightarrow (x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$\Rightarrow x^3+y^3+z^3-3xyz = [0](x^2+y^2+z^2-xy-yz-zx)$

$\Rightarrow x^3+y^3+z^3-3xyz = 0$

$\Rightarrow x^3+y^3+z^3 = 3xyz$ Ans

Q14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3+(7)^3+(5)^3$

Solve $\Rightarrow x+y+z=0$

$\Rightarrow -12+7+5=0$

$\Rightarrow 12-12=0$

$\Rightarrow x+y+z=3xyz$

$\Rightarrow 3(-12)\cdot(7)\cdot(5)$

$\Rightarrow -36 \times 35$

$\Rightarrow -1260$ Ans Let $a=(-12)$; $b=7$, $c=5$.

Here $a+b+c=0$ use the result in Q13.

ii) $(28)^3+(-15)^3+(-13)^3$

Solve $\Rightarrow x+y+z=0$

$\Rightarrow 28+(-15)+(-13) \Rightarrow 28-15-13$

$\Rightarrow 28-28=0$

$\Rightarrow x^3+y^3+z^3=3xyz$

$\Rightarrow 3(28)(-15)(-13)$

$\Rightarrow 84 \times 195$

$\Rightarrow 16,380$ Ans

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Q15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given.

(i) Area : $25a^2 - 35a + 12$

$25 \times 12 = 300$

2	300
2	150
3	75
5	25
5	5
	1

$15 \leftarrow$

Solve ⇒ Or $R = L \times B$

$25a^2 - 15a - 20a + 12$

$5a(5a-3) - 4(5a-3)$

$(5a-3)(5a-4)$

$-15 - 20 = -35$

$L = (5a-3)$

$B = (5a-4)$] Area of Rectangle Ans

(ii) $35y^2 + 13y - 12$

$35 \times 12 = 420$

2	420
2	210
3	105
5	35
7	7
	1

Solve ⇒ $35y^2 + 28y - 15y - 12$

⇒ $7y(5y+4) - 3(5y+4)$

⇒ $(5y+4)(7y-3)$

$28 - 15 = 13$.

$L = (5y+4)$

$B = (7y-3)$] Area of Rectangle Ans

Q16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

Solve ⇒ (i) Volume ⇒ $3x^2 - 12x$

⇒ $3(x^2 - 4x)$

⇒ $3(x)(x-4)$ Ans

Possible Value ⇒ Length = 3, Breadth = (x), Height = $(x-4)$.

(ii) $12Ky^2 + 8Ky - 20K$

Solve ⇒ ⇒ $4K[3y^2 + 2y - 5]$

⇒ $4K[3y^2 + 5y - 3y - 5]$

⇒ $4K[y(3y+5) - 1(3y+5)]$

⇒ $4K(3y+5)(y-1)$ Ans

Possible Value ⇒ Length = $4K$

Breadth = $(3y+5)$

Height = $(y-1)$

The End